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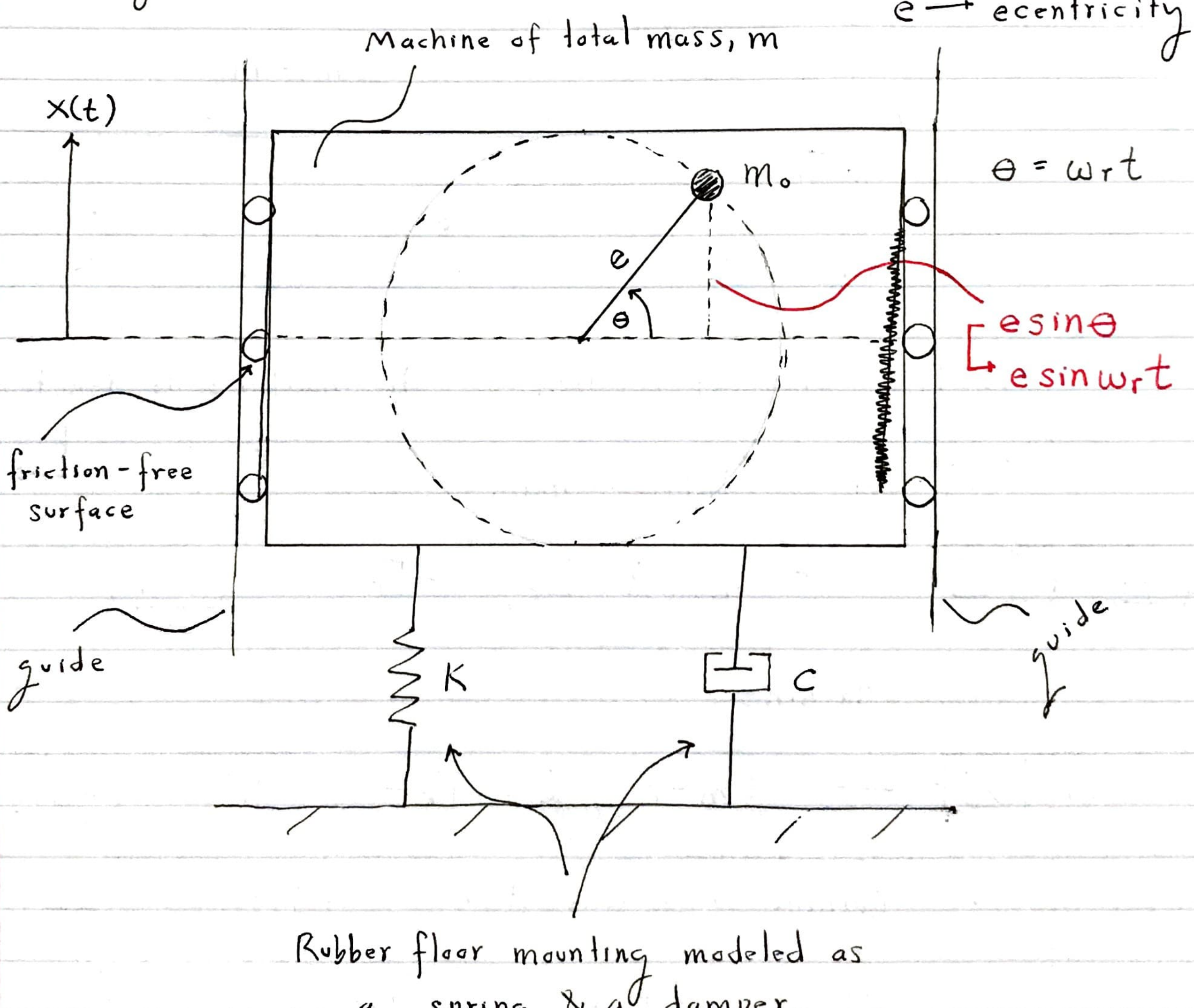
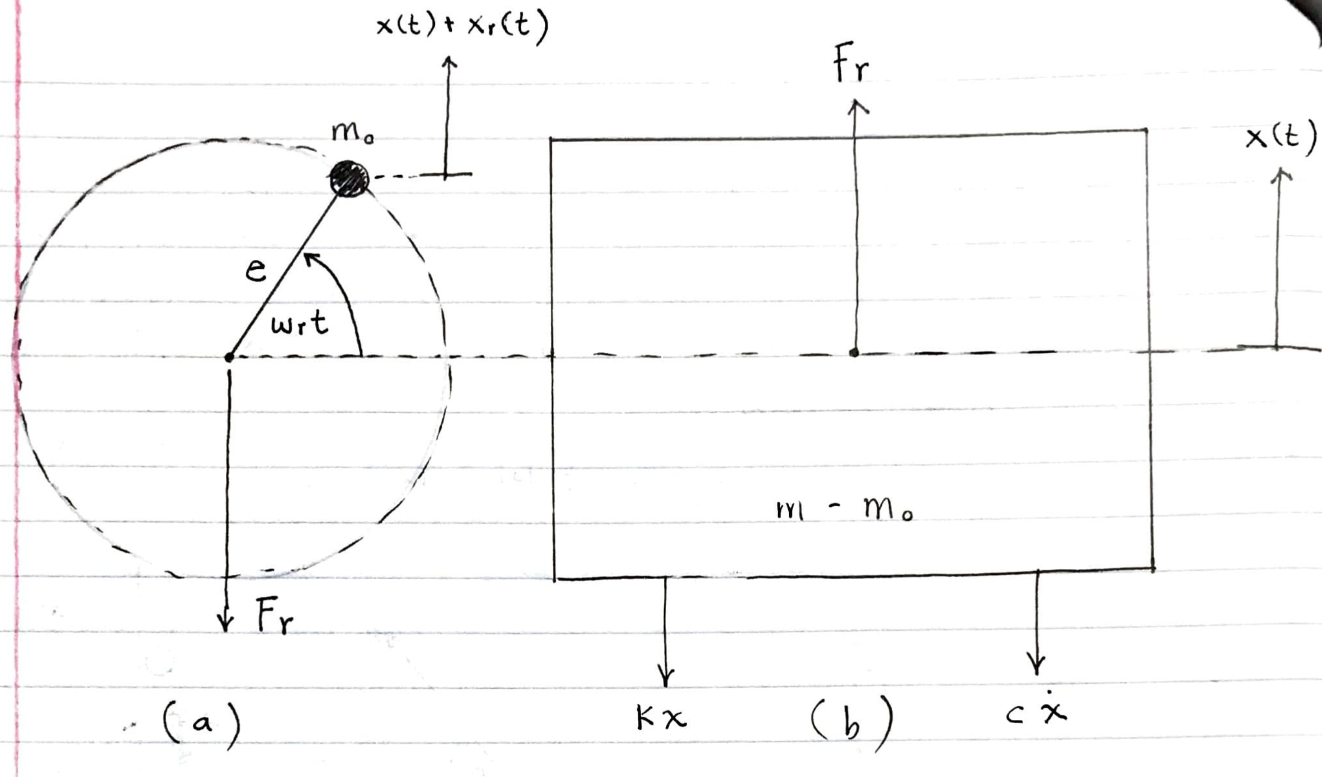
Rotating Unbalance

- Many machines & devices have rotating components, usually driven by electric motors. Small irregularities in the distribution of the mass in the rotating component can cause substantial vibration. This is called a <u>rotating unbalance</u>.

$e \rightarrow$ ecentricity





NOTE: freq. of rotation of the machine is denoted by $\to \omega_r$

(a) out-of-balance mass

~~eq. 1~~) $(+)\uparrow \sum F \longrightarrow \quad m_o(\ddot{x} + \ddot{x}_r) = -F_r \qquad$ (eq. 1)

(b) Machine

$(+)\uparrow \sum F \longrightarrow \quad (m - m_o)\ddot{x} = F_r - c\dot{x} - kx \qquad$ (eq. 2)

... combining equations 1 & 2 yields

$\to \quad m_o(\ddot{x} + \ddot{x}_r) + (m - m_o)\ddot{x} = \cancel{-F_r} + \cancel{F_r} - c\dot{x} - kx$

$$\cancel{m_o\ddot{x}} + m_o\ddot{x}_r + m\ddot{x} - \cancel{m_o\ddot{x}} = -c\dot{x} - kx$$

$$m\ddot{x} + m_o\ddot{x}_r + c\dot{x} + kx = 0 \qquad \text{(eq. 3)}$$

NOTE: forces in the horiz. direction are cancelled by the guides & are not considered here.

Assuming the machine rotates w/ a const. freq., ω_r, the x-comp. of the motion of the mass, m_0, is :

$$\longrightarrow x_r = e \sin \omega_r t \quad \therefore ,$$

$$\ddot{x}_r = - e \omega_r^2 \sin \omega_r t \qquad (\text{eq. } 4)$$

... substituting into eq. 3 yields,

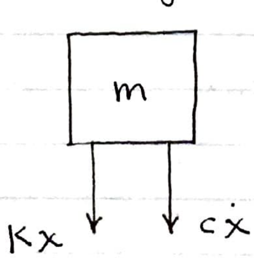
$$\boxed{m\ddot{x} + \cancel{m}\, c\dot{x} + Kx = \underbrace{m_0 e \omega_r^2}_{F_0 = m_0 e \omega_r^2} \sin \omega_r t} \qquad (\text{eq. } 5)$$

$$Kg \cdot m \cdot \frac{rad^2}{s^2} = Kg \cdot \frac{m}{s^2} = N$$

OR

(moving mass, m, up)

FBD:



Dyn. equil. : $\Sigma F = m\ddot{x}$

$$-c\dot{x} - Kx = \cancel{m(\ddot{x} - \ddot{x}_0)}\ (m - m_0)\ddot{x} + \frac{d^2}{dt^2}\left[x + e \sin \omega_r t \right] \cdot m_0$$

NOTE: $x + e \sin \omega_r t$

translation of whole machine

rotation of unbalanced mass

$$-c\dot{x} - kx = \overset{(m-m_o)\ddot{x}}{\cancel{m(\ddot{x}-\ddot{x}_o)}} + \frac{d^2}{dt^2}\left[x + e\sin\omega_r t\right]\cdot m_o$$

$$-c\dot{x} - kx = m\ddot{x} - \cancel{m_o\ddot{x}} + \cancel{m_o\ddot{x}} - m_o e\omega_r^2 \sin\omega_r t$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = m_o e\omega_r^2 \sin\omega_r t}$$

F_o (function of square)

Procedure :

NOTE: $f_o = F_o/m$

particular solution. $\rightarrow X_p = X\sin(\omega_r t - \phi)$

phase $\rightarrow \phi = \tan^{-1}\dfrac{2\zeta r}{1-r^2}$

magnitude $\rightarrow X = \dfrac{m_o e}{m}\cdot\dfrac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$r \rightarrow$ freq. ratio

$\quad r = \dfrac{\omega_r}{\omega_n}$

$\zeta \rightarrow$ damping ratio

$\quad \zeta = \dfrac{c}{2\sqrt{km}}$

$$X_p = X\sin(\omega_r t - \phi) \quad \text{(eq. 6)}$$

$$X = \frac{m_o e}{m}\cdot\frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad \text{(eq. 7)}$$

$$\phi = \tan^{-1}\frac{2\zeta r}{1-r^2} \quad \text{(eq. 8)}$$