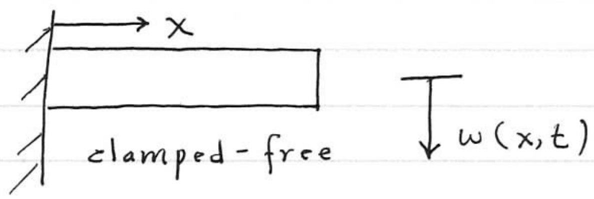
Beam in Transverse Vibration (Bending)



Natural Freq., $\omega_n = \beta_n^2 \sqrt{EI/\rho A}$

where, $\beta_1 l = 1.87510407$

$\sigma_1 = 0.7341$

↳ for mode shapes

Fundamental Freq. $(n=1)$:

$l = 80 \text{ in.}$ (avg. length from branch length measurements)

from website

↳ $E = 2086 \text{ ksi} = 2086 \times 10^3 \text{ lb/in}^2$

↳ $\rho = 51.5 \text{ lb/ft}^3 = 0.0298 \text{ lbm/in}^3$

$I = \dfrac{\pi d^4}{64}$ (where $d = 2.84 \text{ in.}$) $\quad I = \dfrac{\pi (2.84)^4}{64}$

$A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(2.84)^2$

$$\omega_1 = \left(\frac{1.87510407}{80\text{in}}\right)^2 \sqrt{\frac{(2086 \times 10^3 \text{ lb/in}^2)\cdot\left[\pi(2.84\text{in})^4/64\right]}{0.0298 \text{ lbm/in}^3 \cdot \dfrac{\text{lbf}}{\text{lbm}\cdot 32.2\text{ft/s}^2 \cdot 12\text{in/1ft}} \cdot (\pi/4)(2.84\text{in})^2}}$$

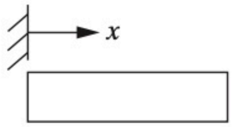
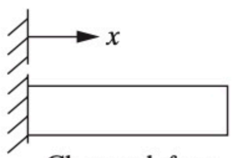
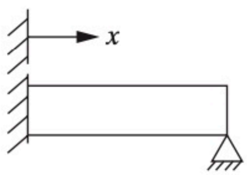
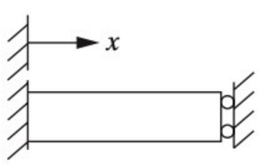
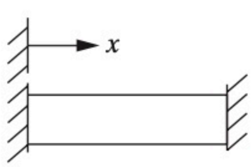
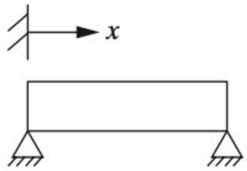
$$\left(\frac{1}{\text{in}}\right)^2 \sqrt{\frac{\cancel{\text{lbf}}}{\cancel{\text{in}^2}} \cdot \cancel{\text{in}^4} \cdot \frac{\text{in}^3}{\text{lbm}} \cdot \cancel{\text{lbm}} \cdot \frac{\cancel{\text{ft}}}{\text{s}^2} \cdot \frac{1}{\cancel{\text{lbf}}} \cdot \frac{\text{in}}{\cancel{\text{ft}}} \cdot \frac{1}{\cancel{\text{in}^2}}}$$

$$\hookrightarrow \left(\frac{1}{\text{in}^2}\right)\sqrt{\frac{\text{in}^4}{\text{s}^2}} = \left(\frac{1}{\cancel{\text{in}^2}}\right) \cdot \frac{\cancel{\text{in}^2}}{\text{s}} = \text{rad/s}$$

$\omega_1 = 64.15 \text{ rad/s}$ $\quad \boxed{\omega_1 = 10.21 \text{ Hz}}$

$$64.15 \frac{\cancel{\text{rad}}}{\text{s}} \times \frac{1 \text{ cycle}}{2\pi \cancel{\text{rad}}} = \underline{10.21 \text{ Hz}}$$

TABLE 6.6 SAMPLE OF VARIOUS BOUNDARY CONFIGURATIONS OF A SLENDER BEAM IN TRANSVERSE VIBRATION OF LENGTH l ILLUSTRATING WEIGHTED NATURAL FREQUENCIES AND MODE SHAPES[a]

Configuration	Weighted frequencies $\beta_n l$ and characteristic equation	Mode shape	σ_n
 Free–free	0 (rigid-body mode) 4.73004074 7.85320462 10.9956078 14.1371655 17.2787597 $\frac{(2n+1)\pi}{2}$ for $n > 5$ $\cos \beta l \cosh \beta l = 1$	$\cosh \beta_n x + \cos \beta_n x$ $-\sigma_n (\sinh \beta_n x + \sin \beta_n x)$[b]	 0.9825 1.0008 0.9999 1.0000 0.9999 1 for $n > 5$
 Clamped–free	1.87510407 4.69409113 7.85475744 10.99554073 14.13716839 $\frac{(2n-1)\pi}{2}$ for $n > 5$ $\cos \beta l \cosh \beta l = -1$	$\cosh \beta_n x - \cos \beta_n x$ $-\sigma_n (\sinh \beta_n x - \sin \beta_n x)$	0.7341 1.0185 0.9992 1.0000 1.0000 1 for $n > 5$
 Clamped–pinned	3.92660231 7.06858275 10.21017612 13.35176878 16.49336143 $\frac{(4n+1)\pi}{4}$ for $n > 5$ $\tan \beta l = \tanh \beta l$	$\cosh \beta_n x - \cos \beta_n x$ $-\sigma_n (\sinh \beta_n x - \sin \beta_n x)$	1.0008 1 for $n > 1$
 Clamped–sliding	2.36502037 5.49780392 8.63937983 11.78097245 14.92256510 $\frac{(4n-1)\pi}{4}$ for $n > 5$ $\tan \beta l + \tanh \beta l = 0$	$\cosh \beta_n x - \cos \beta_n x$ $-\sigma_n (\sinh \beta_n x - \sin \beta_n x)$	0.9825 1 for $n > 1$
 Clamped–clamped	4.73004074 7.85320462 10.9956079 14.1371655 17.2787597 $\frac{(2n+1)\pi}{2}$ for $n > 5$ $\cos \beta l \cosh \beta l = 1$	$\cosh \beta_n x - \cos \beta_n x$ $-\sigma_n (\sinh \beta_n x - \sin \beta_n x)$	0.982502 1.00078 0.999966 1.0000 1.0000 1 for $n > 5$
 Pinned–pinned	$n\pi$ $\sin \beta l = 0$	$\sin \frac{n\pi x}{l}$	none

[a]Here the weighted natural frequencies $\beta_n l$ are related to the natural frequencies by equation (6.101) or $\omega_n = \beta_n^2 \sqrt{EI/\rho A}$, as used in Example 6.5.1. The values of σ_i for the mode shapes are computed from the formulas given in Table 6.5.

[b]There are two free–free mode shapes: $X_0 =$ constant and $X_0 = A(x - l/2)$; the first is translational, the second rotational.