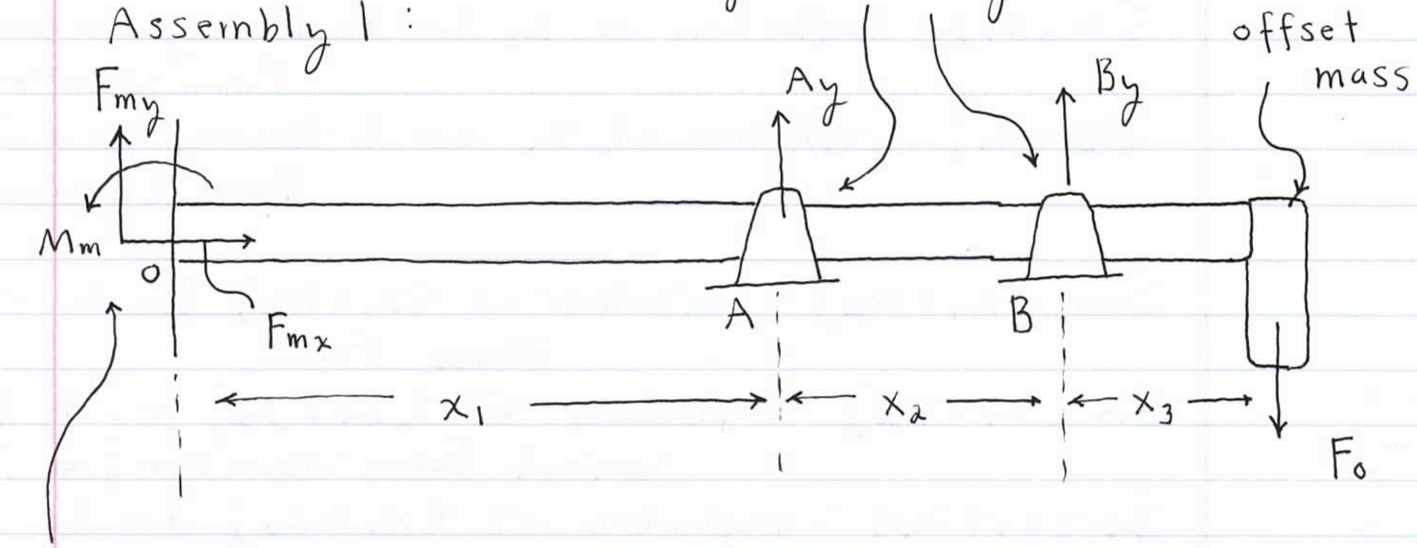
# Pillow Block Forces (Assem. 1)

Assembly 1 :

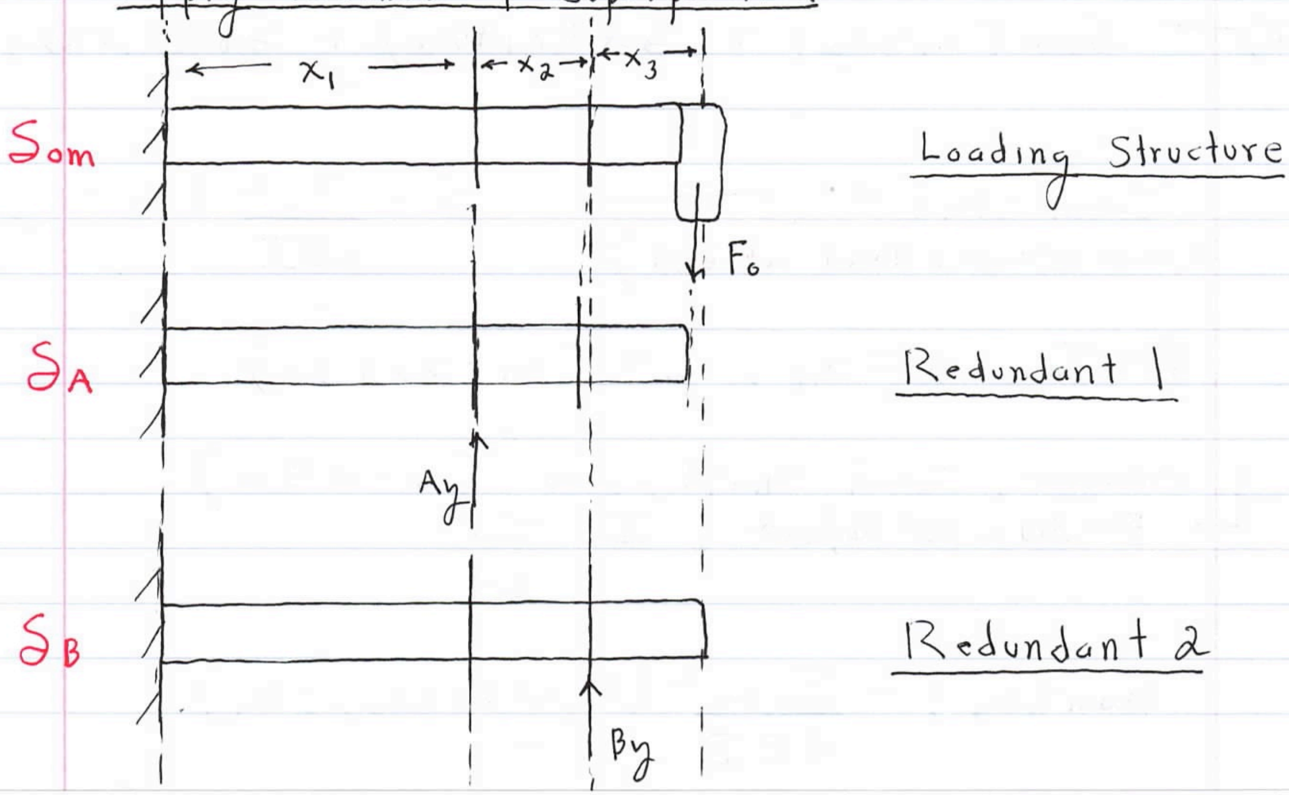



$x_1 = 5 \text{in}$ , $x_2 = 3 \text{in}$ , $x_3 = 3 \text{in}$

for Offset Mass 3 : $F_o(\omega_{n1}) = 19.497 \text{ lbf}$

$\left.\begin{array}{l}\text{\# of Reactions : 5} \\ \text{\# of equilibrium eqns. : 3}\end{array}\right\}$ <u>statically indeterminate.</u>

<u>Apply Method of Superposition</u>



$\delta_{om}(x_1)$ : Deflection at $x_1$ due to Offset Mass Force.
$\delta_A(x_1)$ : Deflection at $x_1$ due to Bearing Vertical Force (Reaction) at A.
$\delta_B(x_1)$ : Deflection at $x_1$ due to Bearing Vertical Force (Reaction) at B.

$\delta_{om}(x_1+x_2)$ : Deflection at $(x_1+x_2)$ due to Offset Mass Force.
$\delta_A(x_1+x_2)$ : Deflection at $(x_1+x_2)$ due to Bearing Vertical Force (Reaction) at A.
$\delta_B(x_1+x_2)$ : Deflection at $(x_1+x_2)$ due to Bearing Vertical Force (Reaction) at B.

(eq.1) $\Sigma \delta(x_1) = 0$

(eq.2) $\Sigma \delta(x_2) = 0$

(eq.1) $\rightarrow$ $\delta_{om}(x_1) + \delta_A(x_1) + \delta_B(x_1) = 0$

(eq.2) $\rightarrow$ $\delta_{om}(x_1+x_2) + \delta_A(x_1+x_2) + \delta_B(x_1+x_2) = 0$

$\delta_{om}(x_1) = \longrightarrow \quad \delta = \dfrac{Px^2}{6EI}(3L - x)$

(concentrated load at end)

$P = F_o, \quad x = x_1, \quad L = (x_1 + x_2 + x_3)$

Circular, Steel Shaft / Rod $(d = 0.5 \text{ in.})$
$\rightarrow$ $E = 30 \times 10^6 \text{ lb/in}^2, \quad I = \dfrac{\pi d^4}{64}$

$$\delta_{om}(x_1) = \frac{F_o x_1^2}{6EI}\left(3(x_1+x_2+x_3) - x_1\right)$$

$$\delta_{om}(x_1) = \frac{-(19.497)(5)^2}{6(30\times10^6)\left(\frac{\pi\cdot 0.5^4}{64}\right)}\cdot(3(11)-5)$$

$$\cancel{lbf}\cdot\cancel{in^2}\cdot\frac{\cancel{in^2}}{\cancel{lbf}}\cdot\frac{1}{\cancel{in^4}}\cdot in = in \qquad \underline{\delta_{om}(x_1) = -24.714\times10^{-3}\,in.}$$

$$\delta_A(x_1) = \longrightarrow \quad \delta = \frac{Px^2}{6EI}(3a-x)$$

(concentrated load at any point)

$P = A_y,\ x = x_1,\ a = x_1,\ E = $ " $30\times10^6 psi$ ", $I = $ " $\frac{\pi d^4}{64}$ "

$$\delta_A(x_1) = \frac{A_y x_1^2}{6EI}(3x_1 - x_1)$$

$$\delta_A(x_1) = \frac{A_y(5)^2}{6(30\times10^6)\left(\frac{\pi\cdot0.5^4}{64}\right)}\cdot(3(5)-5)$$

$$\underline{\delta_A(x_1) = (452.707\times10^{-6}\,in/lbf)\cdot A_y}$$

$$\delta_B(x_1) = \longrightarrow \quad \delta = \frac{Px^2}{6EI}(3a-x)$$

(concentrated load at any point)

$P = B_y,\ x = x_1,\ a = (x_1 + x_2),\ E = $ " ", $I = $ " "

$$\delta_B(x_1) = \cancel{\frac{B_y(x_1)^2}{6(30\times10}} \longrightarrow \frac{B_y x_1^2}{6EI}(3(x_1+x_2)-x_1)$$

$$\delta_B(x_1) = \frac{B_y(5)^2}{6(30\times10^6)\left(\frac{\pi\cdot0.5^4}{64}\right)}\cdot(3(8)-5)$$

$$\underline{\delta_B(x_1) = (860.144\times10^{-6}\,in/lbf)\cdot B_y}$$

(eq. 1) $\longrightarrow$ $(-24.714 \times 10^{-3}\,in) + (452.707 \times 10^{-6}\,in/lbf)\,A_y$
$+ (860.144 \times 10^{-6}\,in/lbf)\,B_y = 0$

$\delta_{om}(x_1 + x_2) = \longrightarrow \quad \delta = \dfrac{P x^2}{6EI}(3L - x)$
(concentrated load at end)

$P = F_o, \quad x = (x_1 + x_2), \quad L = (x_1 + x_2 + x_3), \quad E = \text{" "}, \quad I = \text{" "}$

$$\delta_{om}(x_1 + x_2) = \frac{F_o (x_1 + x_2)^2}{6EI}\left(3(x_1 + x_2 + x_3) - (x_1 + x_2)\right)$$

$$\delta_{om}(x_1 + x_2) = \frac{(-19.497)(8)^2}{6(30 \times 10^6)\left(\frac{\pi \cdot 0.5^4}{64}\right)} \cdot \left(3(11) - (8)\right)$$

$$\underline{\delta_{om}(x_1 + x_2) = -56.489 \times 10^{-3}\,in.}$$

$\delta_A(x_1 + x_2) = \longrightarrow \quad \delta = \dfrac{P a^2}{6EI}(3x - a)$
(concentrated load at any point)

$P = A_y, \quad a = x_1, \quad x = x_1 + x_2, \quad E = \text{" "}, \quad I = \text{" "}$

$$\delta_A(x_1 + x_2) = \frac{A_y x_1^2}{6EI}\left(3(x_1 + x_2) - x_1\right)$$

$$\delta_A(x_1 + x_2) = \frac{A_y (5)^2}{6(30 \times 10^6)\left(\frac{\pi \cdot 0.5^4}{64}\right)} \cdot \left(3(8) - 5\right)$$

$$\underline{\delta_A(x_1 + x_2) = (860.144 \times 10^{-6}\,in/lbf) \cdot A_y}$$

$$\delta_B(x_1+x_2) = \longrightarrow \quad \delta = \frac{Px^2}{6EI}(3a-x)$$

(concentrated load at any point)

$P = B_y$, $x = (x_1+x_2)$, $a = (x_1+x_2)$, $E = $ " ", $I = $ " "

$$\delta_B(x_1+x_2) = \frac{B_y(x_1+x_2)^2}{6EI}\left(3(x_1+x_2)-(x_1+x_2)\right)$$

$$\delta_B(x_1+x_2) = \frac{B_y(8)^2}{6(30\times10^6)\left(\frac{\pi\cdot 0.5^4}{64}\right)}\cdot\left(3(8)-8\right)$$

$$\underline{\delta_B(x_1+x_2) = (1.854\times10^{-3}\ \text{in}/\text{lbf})\cdot B_y}$$

$$(\text{eq. }2) \longrightarrow (-56.489\times10^{-3}\ \text{in.}) + (860.144\times10^{-6}\ \text{in}/\text{lbf})A_y + (1.854\times10^{-3}\ \text{in}/\text{lbf})B_y = 0$$

$$\begin{bmatrix} (452.707\times10^{-6}) & (860.144\times10^{-6}) \\ (860.144\times10^{-6}) & (1.854\times10^{-3}) \end{bmatrix} \begin{bmatrix} A_y \\ B_y \end{bmatrix} = \begin{bmatrix} 24.714\times10^{-3} \\ 56.489\times10^{-3} \end{bmatrix}$$

$$\longrightarrow \underline{A_y = -27.836\ \text{lb}},\ \underline{B_y = 43.383\ \text{lb}}$$

$$+\uparrow \sum F_y = 0;\quad F_{my} + A_y + B_y - F_o = 0$$

$$\hookrightarrow \underline{F_{my} = 3.95\ \text{lb}}$$

$$\underline{M_m = 6.583\ \text{lbin}}$$

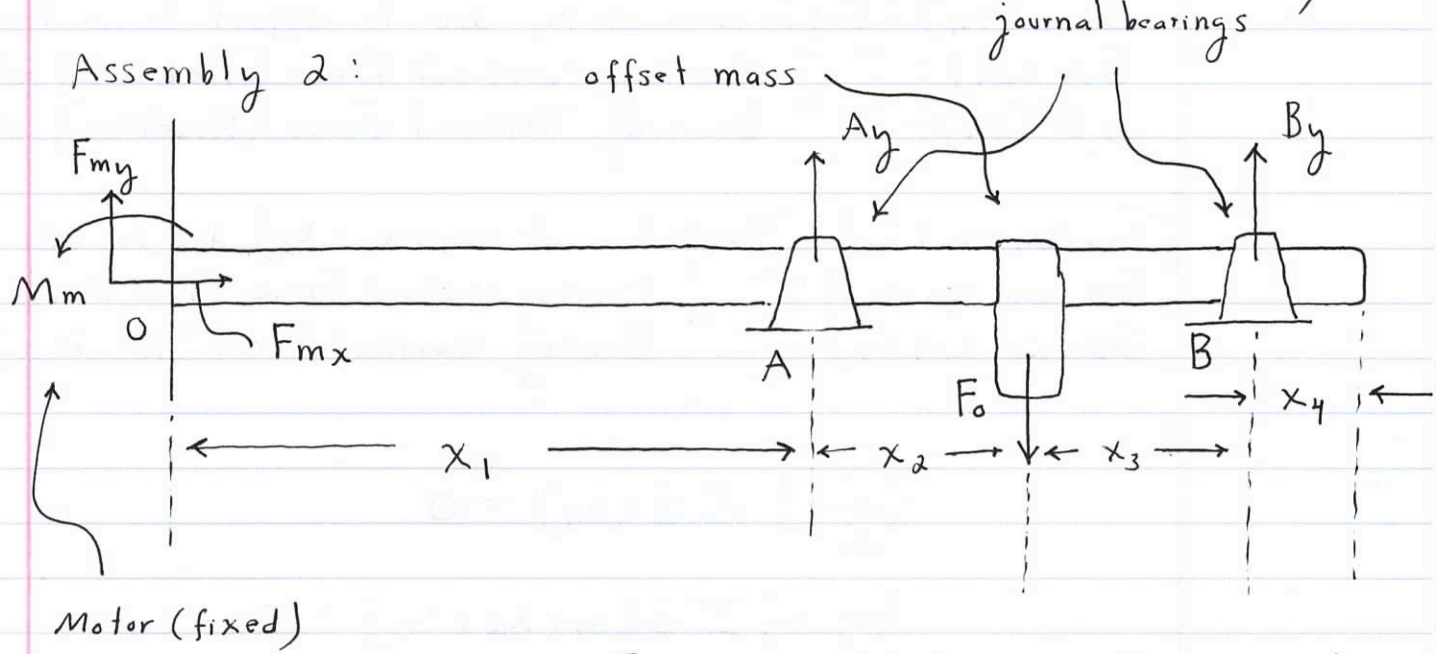
$$\curvearrowleft + \sum M_o = 0;\quad M_m + A_y(x_1) + B_y(x_1+x_2) - F_o(x_1+x_2+x_3) = 0$$

Using MATLAB:

$A_y = -27.729$ lb
$B_y = 43.327$ lb
$F_{my} = 3.899$ lb
$M_m = 6.499$ lbin

# Pillow Block Forces (Assem. 2)

Assembly 2:

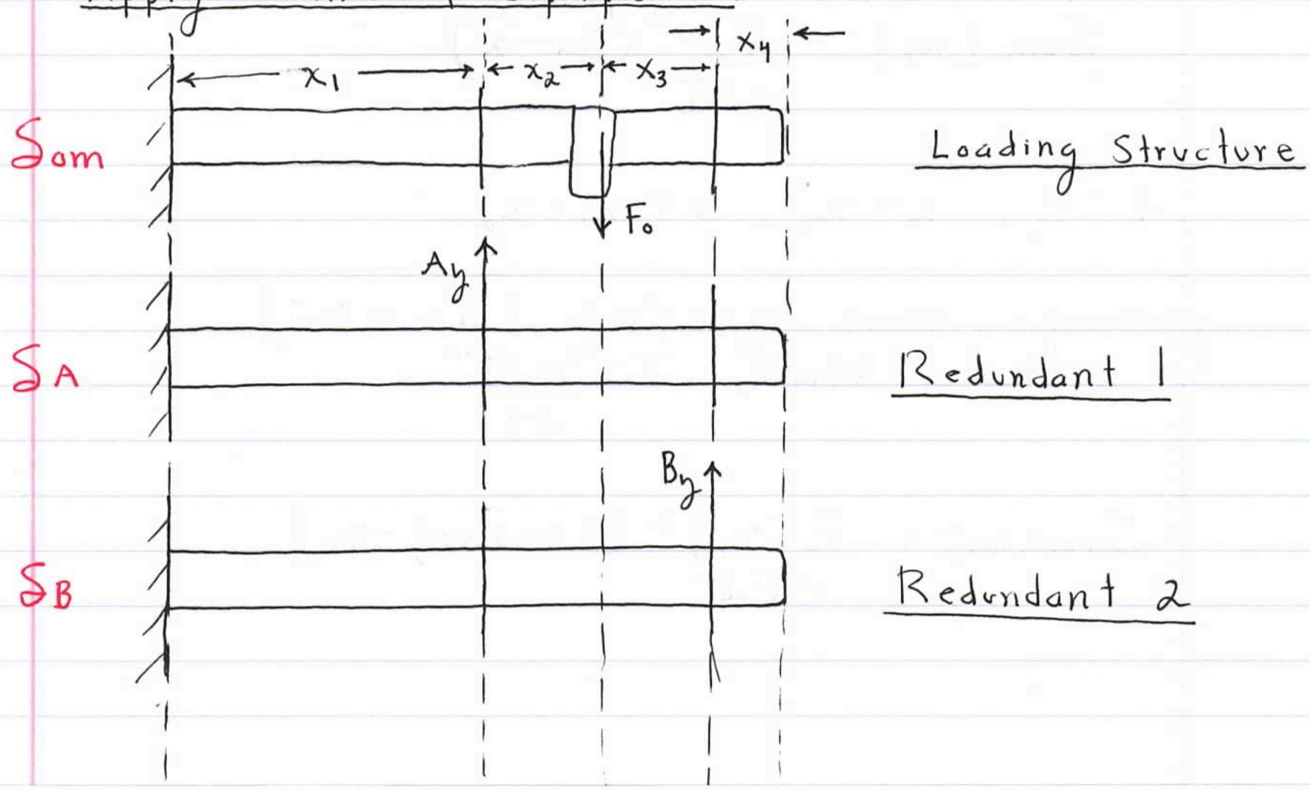



$x_1 = 5\text{ in.}, \quad x_2 = 3\text{ in}, \quad x_3 = 3\text{ in}, \quad x_4 = 1\text{ in.}$

for offset mass 3 : $F_o(\omega_{n_1}) = 19.497\ \text{lbf}$

$\left.\begin{array}{l}\#\text{ of Reactions : } 5 \\ \#\text{ of equilibrium eqns. : } 3\end{array}\right\}$ <u>Statically Indeterminate</u>

<u>Apply Method of Superposition</u>



$\delta_{om}(x_1)$ : "Deflection at $x_1$ due to" Offset Mass Force.
$\delta_A(x_1)$ : " " Bearing Vertical Force (Reaction) at A.
$\delta_B(x_1)$ : " " Bearing Vertical Force (Reaction) at B.

$\delta_{om}(x_1+x_2+x_3)$ : "Deflection at $(x_1+x_2+x_3)$ due to" OM Force.
$\delta_A(x_1+x_2+x_3)$ : " " Bearing Vertical Force (Reaction) at A.
$\delta_B(x_1+x_2+x_3)$ : " " Bearing Vertical Force (Reaction) at B.

$$(\text{eq.}1)\quad \Sigma\,\delta(x_1) = 0$$

$$(\text{eq.}2)\quad \Sigma\,\delta(x_1+x_2+x_3) = 0$$

$$(\text{eq.}1) \rightarrow \delta_{om}(x_1) + \delta_A(x_1) + \delta_B(x_1) = 0$$

$$(\text{eq.}2) \rightarrow \delta_{om}(x_1+x_2+x_3) + \delta_A(x_1+x_2+x_3) + \delta_B(x_1+x_2+x_3) = 0$$

(concentrated load at any point)

$$\delta_{om}(x_1) = \frac{Px^2}{6EI}(3a-x)$$

$P = F_o$, $x = x_1$, $a = x_1 + x_2$

Circular, Steel Shaft / Rod ($d = 0.5$in.)
→ $E =$ "$30\times10^6$ lb/in$^2$", $I =$ "$\frac{\pi d^4}{64}$"

$$\delta_{om}(x_1) = \frac{F_o(x_1)^2}{6EI}\left(3(x_1+x_2) - x_1\right)$$

$$\delta_{om}(x_1) = \frac{(-19.497)(5)^2}{6(30\times10^6)\left(\frac{\pi\cdot 0.5^4}{64}\right)}\cdot(3(8)-5)$$

$$\cancel{lbf}\cdot\cancel{in^2}\cdot\frac{\cancel{in^2}}{\cancel{lbf}}\cdot\frac{1}{\cancel{in^4}}\cdot in = in \qquad \underline{\delta_{om}(x_1) = -16.770\times10^{-3}\ in.}$$

(concentrated load at any pt.)

$$\delta_A(x_1) = \frac{Px^2}{6EI}\cdot(3a-x)$$

$P = A_y$, $x = x_1$, $a = x_1$, $E =$ " ", $I =$ " "

$$\delta_A(x_1) = \frac{A_y x_1^2}{6EI}(3x_1 - x_1)$$

$$\delta_A(x_1) = \frac{A_y(5)^2}{6(30\times10^6)\left(\frac{\pi\cdot 0.5^4}{64}\right)}(3(5)-5)$$

$$\underline{\delta_A(x_1) = (452.707\times10^{-6}\ in/lbf)\cdot A_y}$$

(concentrated load at any pt.)

$$\delta_B(x_1) = \frac{Px^2}{6EI}(3a-x)$$

$P = B_y$, $x = x_1$, $a = x_1 + x_2 + x_3$, $E =$ " ", $I =$ " "

$$\delta_B(x_1) = \frac{B_y x_1^2}{6EI}(3(x_1+x_2+x_3)-x_1)$$

$$\delta_B(x_1) = \frac{B_y(5)^2}{6(30\times10^6)\left(\frac{\pi\cdot 0.5^4}{64}\right)}\cdot(3(11)-5)$$

$$\underline{\delta_B(x_1) = (1.268 \times 10^{-3} \text{ in/lbf})\, B_y}$$

(eq. 1) → $$(-16.770 \times 10^{-3} \text{ in.}) + (452.707 \times 10^{-6} \text{ in/lbf})\, A_y + (1.268 \times 10^{-3} \text{ in/lbf})\, B_y = 0$$

(concentrated load at any pt.)

$$\delta_{om}(x_1 + x_2 + x_3) = \frac{Pa^2}{6EI}(3x - a)$$

$P = F_o$, $a = x_1 + x_2$, $x = x_1 + x_2 + x_3$, E = " ", I = " "

$$\delta_{om}(x_1 + x_2 + x_3) = \frac{F_o (x_1 + x_2)^2}{6EI}\left(3(x_1 + x_2 + x_3) - (x_1 + x_2)\right)$$

$$\delta_{om}(x_1 + x_2 + x_3) = \frac{(-19.497)(8)^2}{6(30 \times 10^6)\left(\frac{\pi \cdot 0.5^4}{64}\right)} \cdot (3(11) - (8))$$

$$\underline{\delta_{om}(x_1 + x_2 + x_3) = -56.489 \times 10^{-3} \text{ in.}}$$

(concentrated load @ any pt.)

$$\delta_A(x_1 + x_2 + x_3) = \frac{Pa^2}{6EI}(3x - a)$$

$P = A_y$, $a = x_1$, $x = x_1 + x_2 + x_3$, E = " ", I = " "

$$\delta_A(x_1 + x_2 + x_3) = \frac{A_y x_1^2}{6EI}\left(3(x_1 + x_2 + x_3) - x_1\right)$$

$$\underline{\delta_A(x_1 + x_2 + x_3) = (1.268 \times 10^{-3} \text{ in/lbf})\, A_y}$$

$$\delta_A(x_1 + x_2 + x_3) = \frac{A_y (5)^2}{6(30 \times 10^6)\left(\frac{\pi \cdot 0.5^4}{64}\right)} \cdot (3(11) - 5)$$

(concentrated load at any pt.)

$$\delta_B(x_1+x_2+x_3) = \frac{Px^2}{6EI}(3a-x)$$

$P = B_y$, $x = x_1+x_2+x_3$, $a = x_1+x_2+x_3$, $E = $ " ", $I = $ " "

$$\delta_B(x_1+x_2+x_3) = \frac{B_y(x_1+x_2+x_3)^2}{6EI}\left(3(x_1+x_2+x_3)-(x_1+x_2+x_3)\right)$$

$$\delta_B(x_1+x_2+x_3) = \frac{B_y(11)^2}{6(30\times10^6)\left(\frac{\pi\cdot0.5^4}{64}\right)}\cdot(3(11)-11)$$

$$\underline{\delta_B(x_1+x_2+x_3) = (4.820\times10^{-3}\ \text{in/lbf})\,B_y}$$

(eq. 2) $\rightarrow$ $(-56.489\times10^{-3}\ \text{in}) + (1.268\times10^{-3}\ \text{in/lbf})\,A_y + (4.820\times10^{-3}\ \text{in/lbf})\,B_y = 0$

$$\begin{bmatrix} (452.707\times10^{-6}) & (1.268\times10^{-3}) \\ (1.268\times10^{-3}) & (4.820\times10^{-3}) \end{bmatrix}\begin{bmatrix} A_y \\ B_y \end{bmatrix} = \begin{bmatrix} 16.770\times10^{-3} \\ 56.489\times10^{-3} \end{bmatrix}$$

$\longrightarrow$ $\underline{A_y = 16.027\ \text{lb}}$, $\underline{B_y = 7.503\ \text{lb}}$

$+\uparrow \Sigma F_y = 0;\quad F_{my} + A_y + B_y - F_o = 0$

$\rightarrow$ $\underline{F_{my} = -4.033\ \text{lb}}$

$\underline{M_m = -6.692\ \text{lbin}}$

$\circlearrowleft + \Sigma M_o = 0;\quad M_m + A_y(x_1) - F_o(x_1+x_2) + B_y(x_1+x_2+x_3) = 0$

Using MATLAB:

$A_y = 16.048$ lb
$B_y = 7.499$ lb
$F_{my} = -4.049$ lb
$M_m = -6.749$ lb in